CHAPITRE XX : DÉVELOPPEMENTS LIMITÉS

Correction

a) On a

$$\begin{aligned}(\ln(1+x))^2 &= \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^5}{5} + \underset{x\to 0}{o}(x^4)\right)\left(x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \underset{x\to 0}{o}(x^4)\right) \\ &= x^2 - x^3 + \frac{11}{12}x^4 - \frac{5}{6}x^5 + \underset{x\to 0}{o}(x^5) = x^2\left(1 - x + \frac{11}{12}x^2 - \frac{5}{6}x^3 + \underset{x\to 0}{o}(x^3)\right).\end{aligned}$$

Dès lors,

$$\frac{1}{(\ln(1+x))^2} = \frac{1}{x^2}\frac{1}{1-\left(x - \frac{11}{12}x^2 + \frac{5}{6}x^3 + \underset{x\to 0}{o}(x^3)\right)}.$$

En posant $u = x - \frac{11}{12}x^2 + \frac{5}{6}x^3 + \underset{x\to 0}{o}(x^3)$, on a $u^2 = x^2 - \frac{11}{6}x^3 + \underset{x\to 0}{o}(x^3)$, $u^3 = x^3 + \underset{x\to 0}{o}(x^3)$ et $\underset{u\to 0}{o}(u^3) = \underset{x\to 0}{o}(x^3)$. En utilisant le développement limité en 0 $\frac{1}{1-u} = 1 + u + u^2 + u^3 + \underset{u\to 0}{o}(u^3)$, on obtient

$$\begin{aligned}\frac{1}{(\ln(1+x))^2} &= \frac{1}{x^2}\left[1 + x - \frac{11}{12}x^2 + \frac{5}{6}x^3 + x^2 - \frac{11}{6}x^3 + x^3 + \underset{x\to 0}{o}(x^3)\right] \\ &= \frac{1}{x^2}\left[1 + x + \frac{1}{12}x^2 + \underset{x\to 0}{o}(x^3)\right] = \frac{1}{x^2} + \frac{1}{x} + \frac{1}{12} + \underset{x\to 0}{o}(x).\end{aligned}$$

b) On a

$$\begin{aligned}&\sqrt{x+2x^3} \\ =&\sqrt{x}\sqrt{1+2x^2} = \sqrt{x}\left(1 + \frac{2x^2}{2} - \frac{(2x^2)^2}{8} + \underset{x\to 0}{o}(x^4)\right) = x^{\frac{1}{2}}\left(1 + x^2 - \frac{x^4}{2} + \underset{x\to 0}{o}(x^4)\right) \\ =&x^{\frac{1}{2}} + x^{\frac{5}{2}} - \frac{1}{2}x^{\frac{9}{2}} + \underset{x\to 0}{o}(x^{\frac{9}{2}}).\end{aligned}$$