

CHAPITRE XII : CALCUL MATRICIEL ET SYSTÈMES LINÉAIRES

Correction

On a

$$\begin{array}{l}
\begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix} \underset{L}{\sim} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix} \underset{L_1 \leftrightarrow L_2}{\sim} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \underset{L}{\sim} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\underset{L}{\sim} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix} \underset{L_3 \leftarrow L_3 - 2L_2}{\sim} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \underset{L}{\sim} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix} \\
\underset{L}{\sim} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \underset{L_3 \leftarrow -L_3}{\sim} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix} \underset{L}{\sim} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix} \\
\underset{L}{\sim} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \underset{\substack{L_1 \leftarrow L_1 - 2L_3 \\ L_2 \leftarrow L_2 - 2L_3}}{\sim} \begin{pmatrix} -4 & 1 & 2 \\ -3 & 0 & 2 \\ 2 & 0 & -1 \end{pmatrix} \underset{L}{\sim} \begin{pmatrix} -4 & 1 & 2 \\ -3 & 0 & 2 \\ 2 & 0 & -1 \end{pmatrix} \\
\underset{L}{\sim} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \underset{L_1 \leftarrow L_1 - L_2}{\sim} \begin{pmatrix} -1 & 1 & 0 \\ -3 & 0 & 2 \\ 2 & 0 & -1 \end{pmatrix} \underset{L}{\sim} \begin{pmatrix} -1 & 1 & 0 \\ -3 & 0 & 2 \\ 2 & 0 & -1 \end{pmatrix}
\end{array}$$

Par conséquent,

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ -3 & 0 & 2 \\ 2 & 0 & -1 \end{pmatrix}.$$