

## CHAPITRE III : ENSEMBLES, APPLICATIONS ET RELATIONS

## Correction

a) On a

$$\begin{aligned} A\Delta A &= (A \cap \bar{A}) \cup (\bar{A} \cap A) = \emptyset \cup \emptyset = \emptyset, \\ A\Delta\emptyset &= (A \cap \bar{\emptyset}) \cup (\bar{A} \cap \emptyset) = (A \cap E) \cup (\bar{A} \cap \emptyset) = A \cup \emptyset = A, \\ A\Delta E &= (A \cap \bar{E}) \cup (\bar{A} \cap E) = (A \cap \emptyset) \cup (\bar{A} \cap E) = \emptyset \cup \bar{A} = \bar{A}. \end{aligned}$$

b) On a  $\bar{A}\Delta\bar{B} = (\bar{A} \cap \overline{\bar{B}}) \cup (\overline{\bar{A}} \cap \bar{B}) = (\bar{A} \cap B) \cup (A \cap \bar{B}) = (A \cap \bar{B}) \cup (\bar{A} \cap B) = A\Delta B$ .

c) Soit  $(A, B, C, D)$  un quadruplet de parties de  $E$ . On a

$$\begin{aligned} (A \cap B)\Delta(C \cap D) &= [(A \cap B) \cap \overline{(C \cap D)}] \cup [\overline{(A \cap B)} \cap (C \cap D)] \\ &= [(A \cap B) \cap (\bar{C} \cup \bar{D})] \cup [(\bar{A} \cup \bar{B}) \cap (C \cap D)] \\ &= \underbrace{(A \cap B \cap \bar{C})}_{\subset A \cap \bar{C}} \cup \underbrace{(A \cap B \cap \bar{D})}_{\subset B \cap \bar{D}} \cup \underbrace{(\bar{A} \cap C \cap D)}_{\subset \bar{A} \cap C} \cup \underbrace{(\bar{B} \cap C \cap D)}_{\subset \bar{B} \cap D} \\ &\subset (A \cap \bar{C}) \cup (\bar{A} \cap C) \cup (B \cap \bar{D}) \cup (\bar{B} \cap D) = (A\Delta C) \cup (B\Delta D). \end{aligned}$$