

## CHAPITRE II : NOMBRES COMPLEXES

## Correction

On a

$$\begin{aligned} A_n &= \sum_{k=0}^n \binom{n}{k} \cos(kx) = \sum_{k=0}^n \binom{n}{k} \operatorname{Re}(e^{ikx}) = \operatorname{Re} \left[ \sum_{k=0}^n \binom{n}{k} e^{ikx} \right] \\ &= \operatorname{Re} \left[ \sum_{k=0}^n \binom{n}{k} (e^{ix})^k 1^{n-k} \right] \\ &= \operatorname{Re} \left[ (1 + e^{ix})^n \right] \text{ grâce à la formule du binôme de Newton} \\ &= \operatorname{Re} \left[ e^{i\frac{nx}{2}} (e^{-i\frac{x}{2}} + e^{i\frac{x}{2}})^n \right] = \operatorname{Re} \left[ e^{i\frac{nx}{2}} \left( 2 \cos \frac{x}{2} \right)^n \right] \\ &= 2^n \cos^n \left( \frac{x}{2} \right) \operatorname{Re} \left[ e^{i\frac{nx}{2}} \right] = 2^n \cos^n \left( \frac{x}{2} \right) \cos \left( \frac{nx}{2} \right). \end{aligned}$$