CHAPITRE II : NOMBRES COMPLEXES
Correction

On a $|-1+i| = \sqrt{2}$ donc
$$-1+i = \sqrt{2}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \sqrt{2}e^{i\frac{3\pi}{4}}.$$

Ainsi, en écrivant z sous forme trigonométrique $z = |z|e^{i\theta}$, on obtient

$$\begin{aligned} z^3 = -1+i \Leftrightarrow |z|^3 e^{3i\theta} = \sqrt{2}e^{i\frac{3\pi}{4}} \Leftrightarrow \begin{cases} |z|^3 = \sqrt{2} \\ 3\theta \equiv \frac{3\pi}{4} [2\pi] \end{cases} & \Leftrightarrow \begin{cases} |z| = 2^{\frac{1}{6}} \\ \exists k \in \mathbb{Z}, \quad 3\theta = \frac{3\pi}{4} + 2k\pi \end{cases} \\ \Leftrightarrow \begin{cases} |z| = 2^{\frac{1}{6}} \\ \exists k \in \mathbb{Z}, \quad \theta = \frac{\pi}{4} + \frac{2k\pi}{3} \end{cases} & \\ \Leftrightarrow \exists k \in [\![0, 2]\!], \quad z = 2^{\frac{1}{6}} e^{i\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right)}. & \end{aligned}$$

L'ensemble des racines cubiques de $-1+i$ est donc l'ensemble $\left\{2^{\frac{1}{6}} e^{i\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right)}, \quad k \in [\![0, 2]\!]\right\}$.